2014 EAAC in TAIPEI TAIWAN

18TH EAST ASIAN ACTUARIAL CONFERENCE

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Taipei International Convention Center in Taipei Taiwan

Mortality Improvement for Some Regions – Methodology and Analysis

18th EAAC

Taipei Taiwan

October 15, 2014

Actuarial Institute of Chinese Taipei A. I. C. T.

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Agenda

Mortality Improvement Methods

Lee Carter Model

Model fit and Analysis

Result

Long term vs. Short-term

Population data vs. insurance data

Deduction Factors

Non-Parametric

parametric

uncertainty

Applications

1. SOA
2. CMIB
3. Other Actuarial Methods
4. Lee-Carter

SOA Methods

$$q_x^{1994+n} = q_x^{1994} \cdot (1 - AA_x)^n$$

This method will have a base rate q^{1994} and the reduction factor AA_x for each age x.

AA_x was obtained:

- Data Source
  - CSRS for age 25-65 for 1987-93
  - add SSA for age 1-24 and 60-120 for 1977-93
- Average Trends
  - Linear Regression of $\log(m_{x,t})$ 5-year age group for data CSRS and SSA for1987-1993 and 1977-1993 respectively

CMIB Methods

$$RF(x,t) = \alpha(x) + [1-\alpha(x)] \cdot [1-f_n(x)]^{t/n}$$

$$q_{x,t} = q_{x,0} \cdot RF(x,t)$$

RF(*x,t*) - Exponential Decay Characterized by two age-dependent **parameters**. α(x) denotes the value to be asymptotically approached when t ends to infinite, while f_n is the percentage of the total fall (1- α(x)) assumed to occur in n years.

Two set of tables 80 and 92 series (1979-82 and 1991-94 experiences, respectively) for annuitants and pensioners.

80 Series - n was fixed at value of 20 and f_{20} at 0.6 for all ages. And α(x) is expressed as:

$$\alpha(x) = \begin{cases} 0.5, & x < 60 \\ \dfrac{x-10}{100}, & 60 \le x \le 110 \\ 1, & x > 110 \end{cases}$$

CMIB Methods

$$RF(x,t) = \alpha(x) + [1-\alpha(x)] \cdot [1 - f_n(x)]^{t/n}$$

$$q_{x,t} = q_{x,0} \cdot RF(x,t)$$

92 Series - n remained fixed at 20 but 1-f_{20} values linearly from 0.45 to 0.71 between ages 60 to 110, below 60 and above 110, constant values with the values already mentioned apply. And α(x) and fn are expressed as the following:

$$\alpha(x) = \begin{cases} 0.13, & x < 60 \\ 1 + 0.87 \cdot \dfrac{x-110}{50}, & 60 \le x \le 110 \\ 1, & x > 110 \end{cases}$$

$$f_{20}(x) = \begin{cases} 0.55, & x < 60 \\ \dfrac{(110-x)\cdot 0.55 + (x-60)\cdot 0.29}{50}, & 60 \le x \le 110 \\ 0.29, & x > 110 \end{cases}$$

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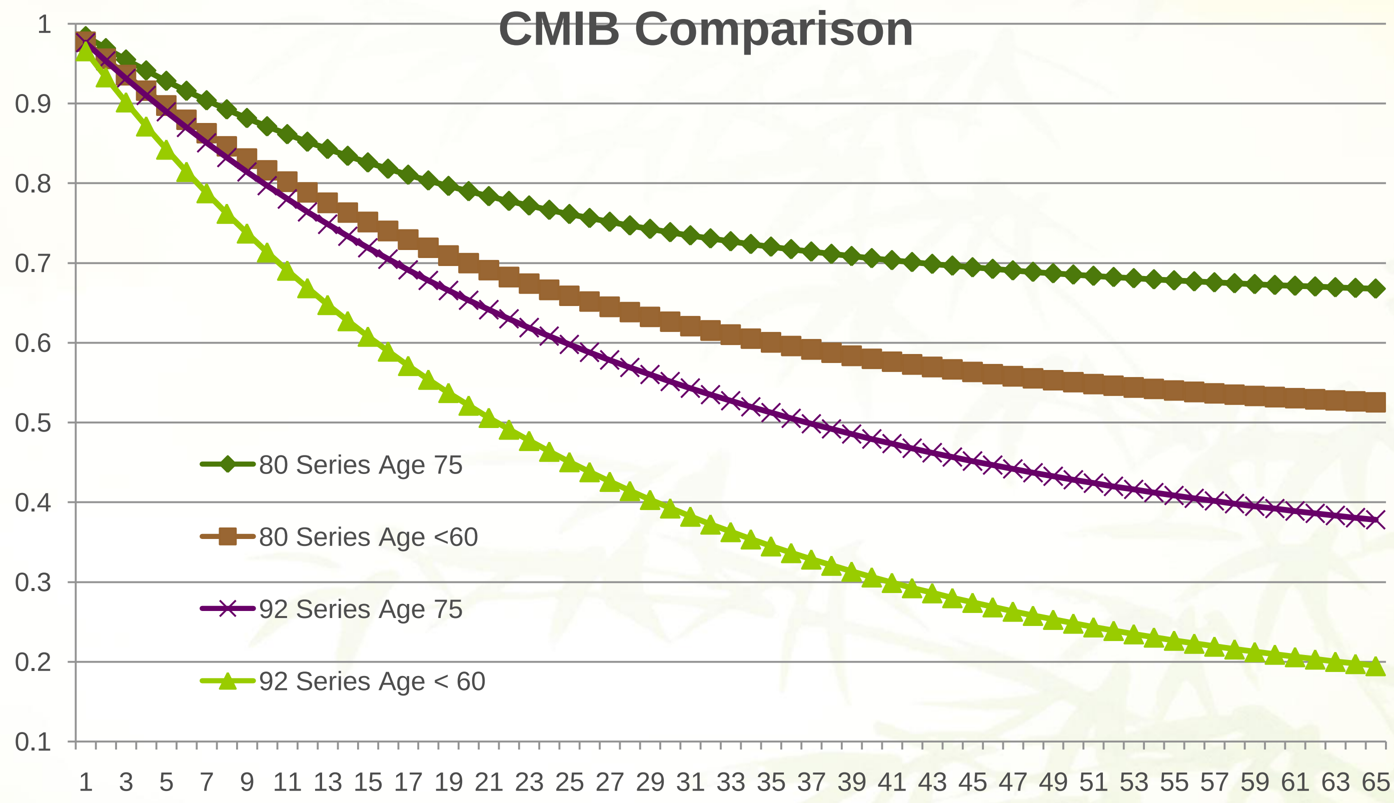
CMIB Methods



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Other Actuarial Method:

Determine "Base" rate

Most Recent Mortality Rate

Define Factor

- Ratio of the mortality to the previous year's mortality
- Linear regression

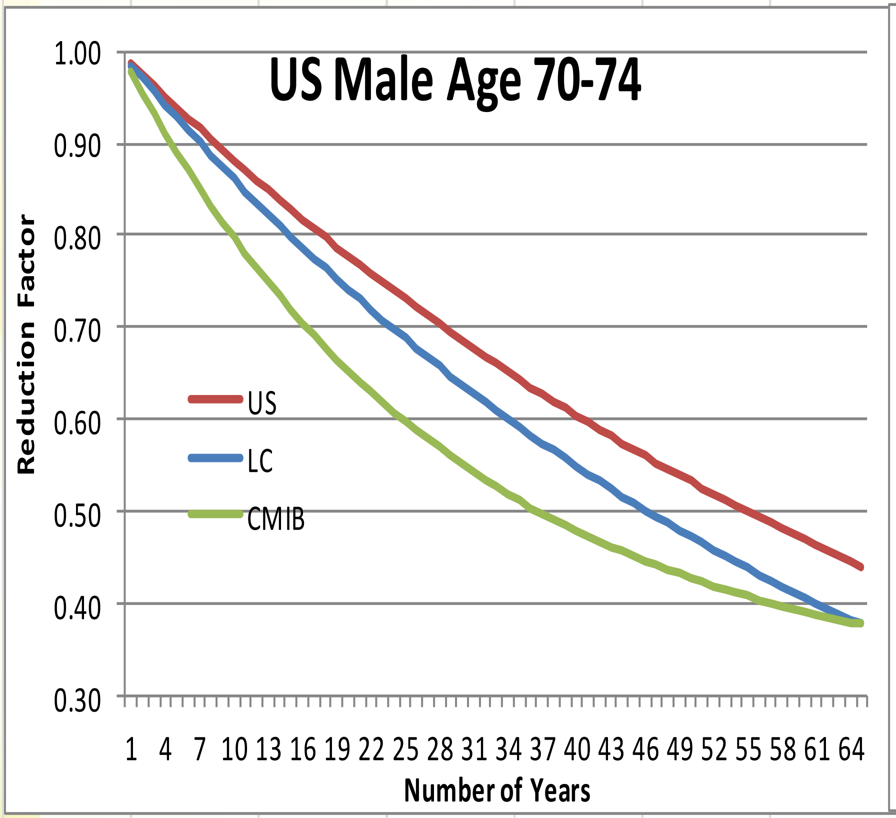
Methods

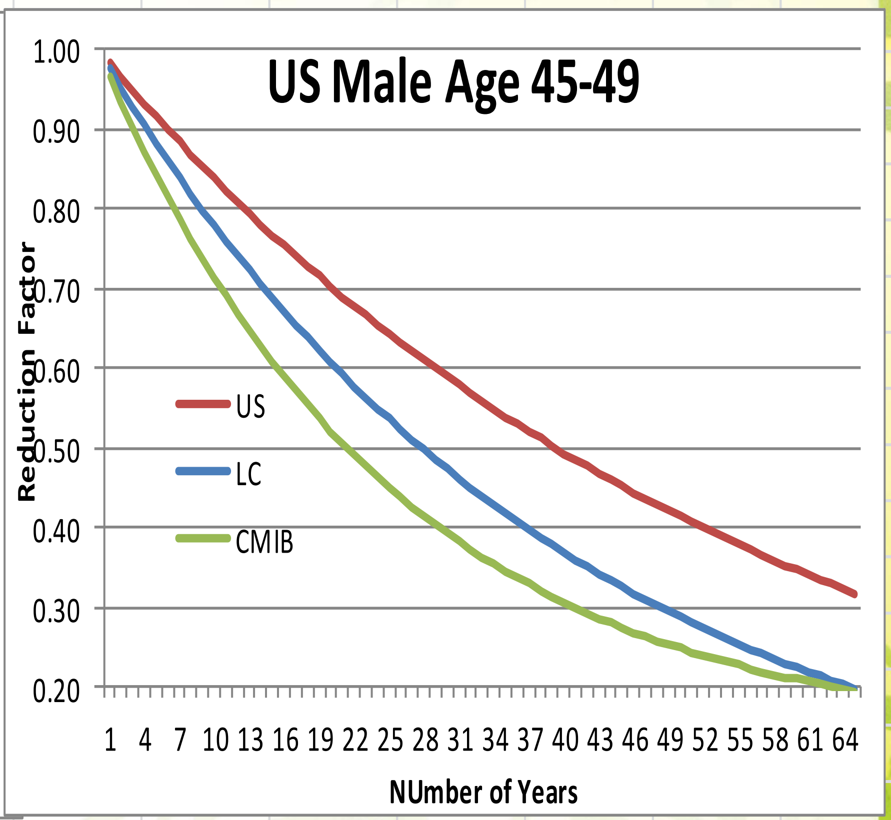
- Arithmetic average
- Mean
- Geometric average
- Weighted average

Lee-Carter's Extrapolating Methodology overview

- Project age pattern and measure uncertainty
- SVD to Solve the age-specific parameters as well as the preliminary mortality index
- ARIMA (0,1,0) a random walk with a drift to project mortality index

The reduction factor Comparison for the SOA, CMIB, and LC methodology





Data

Model

Model fit and Analysis

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Date Source

Region	CalYr	# of CalYr	ProjPeriod	# of ProjYr
Japan	1947-2012	66	1973-2012	40
Taiwan	1970-2010	41	1970-2010	41
USA	1933-2010	78	1971-2010	40

HMD: Human Mortality Database is a population data that it is administrated by UC Berkley

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General Model

$$\ln(m_{x,t}) = a_x + b_x \cdot k_t + \varepsilon_{x,t} \qquad (1)$$

Where, $m_{x,t}$ is the central mortality rate for age x for year t; a_x and b_x are parameters dependent only on age x; k_t is factor to be modeled as a time series; and the $\varepsilon_{x,t}$ error term, is assumed to have mean zero and standard deviation σ_ε.

Re-Write Model as

$$m_{x,t} = e^{a_x + b_x k_t} \qquad (2)$$

Where, e^{a_x} is the general shape across age of the mortality schedule; the b_x profile tells us which rates decline rapidly and which rates decline slowly in response to changes in k_t

$$\left(\frac{d \ln m_{x,t}}{dt} = b_x \cdot \frac{dk_t}{dt} \right)$$

The model (1) cannot be fitted by simple regression methods;

It allows for several solutions.

To deal the above,

Use SVD, and

b_x and k_t is normalized to sum to unity and to zero respectively.

SVD overview

- SVD Let A be order nxm, then there are unitary matrices U and V, of order n and m respectively, such that $A=UFV^*$, where F is a rectangular diagonal matrix of order mxn,

$$F = \begin{bmatrix} \mu_1 & & 0 & \\ & \ddots & & \\ & & \mu_r & \\ 0 & & 0 & \\ & & & \ddots \end{bmatrix}$$

- With $F_{ii} = \mu_i$. The numbers μ_i are called the singular values of A. They are all real and positive, and they can be arranged so that

$$\mu_1 \geq \mu_2 \geq \cdots \geq \mu_r > 0$$

- Where r is the rank of the matrix.
- V* is a conjugate transpose.

SVD applied to $\ln(m_{x,t})$,

$$\ln(m_{x,t}) = U_{n\times n} F V'_{T\times T} \qquad (3)$$

Or

$$\ln(m_{x,t}) = \sum_{i=1}^{r} \mu_i \cdot u_i(x) \cdot v_i(t) \qquad (4)$$

Note that, V (and also U) is real number, so the conjugate transpose V* is equal to the transpose V'.

First 10 Singular values

	Male		
SV	Japan	Taiwan	US
μ_1	164.702	151.638	152.214
μ_2	3.054	3.759	2.418
μ_3	0.832	2.110	0.931
μ_4	0.586	1.321	0.718
μ_5	0.389	0.792	0.393
μ_6	0.365	0.539	0.300
μ_7	0.298	0.440	0.219
μ_8	0.233	0.354	0.170
μ_9	0.202	0.289	0.129
μ_{10}	0.178	0.268	0.110

	Female		
SV	Japan	Taiwan	US
μ_1	180.280	166.811	167.934
μ_2	4.257	2.971	2.012
μ_3	0.801	1.518	0.848
μ_4	0.452	0.790	0.672
μ_5	0.397	0.663	0.352
μ_6	0.320	0.533	0.249
μ_7	0.285	0.458	0.220
μ_8	0.205	0.372	0.174
μ_9	0.176	0.333	0.129
μ_{10}	0.165	0.268	0.102

The first singular value is larger for all three regions

Use the first singular value component

$$\ln(m_{x,t}) = \mu_1 u_1(x) \cdot v_1(t) + \varepsilon_{x,t} \qquad (5)$$

The portion of the total temporal variances explained by the first SV component is all over 95% (except for Taiwan), that seemed captured important of data.

Male			Female		
Japan	Taiwan	US	Japan	Taiwan	US
0.9596	0.9311	0.9632	0.9587	0.9458	0.9699
0.9774	0.9542	0.9785	0.9814	0.9626	0.9816

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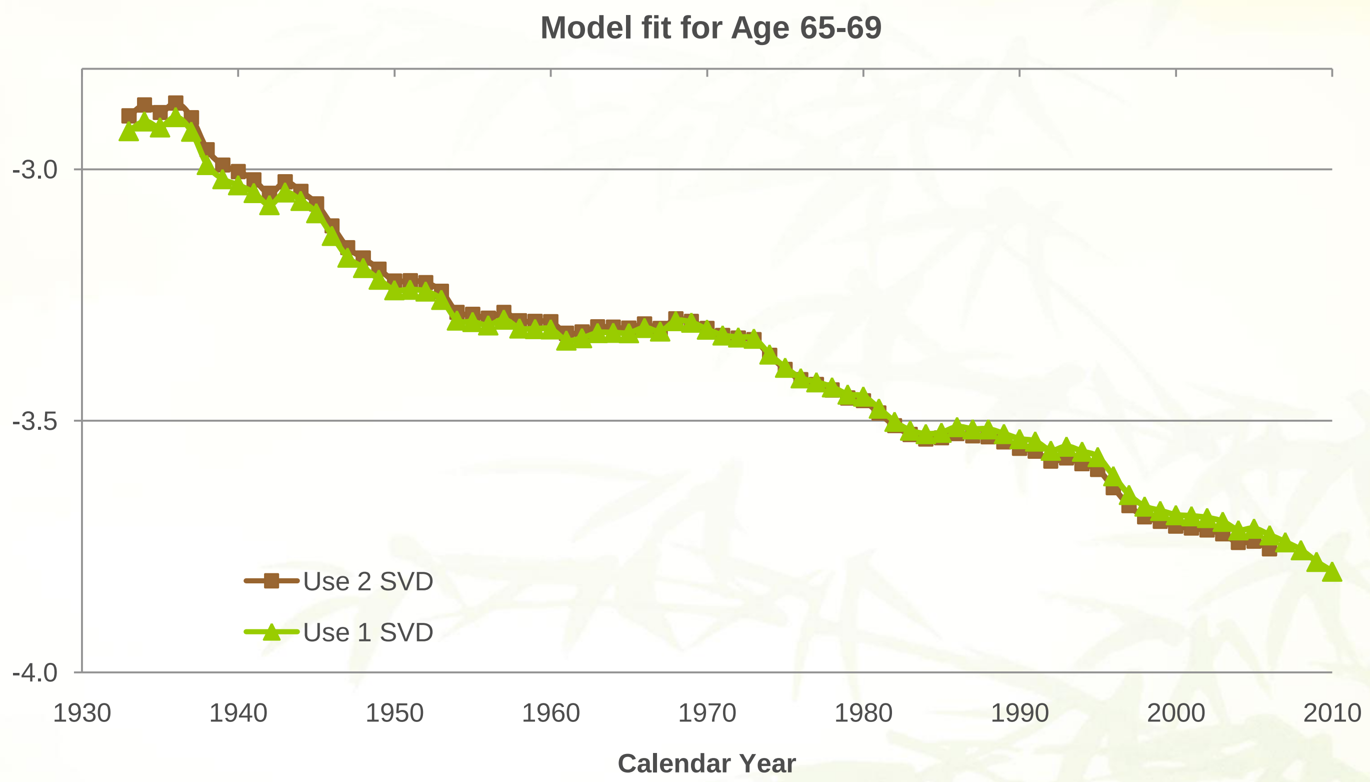
Use one SV (μ_1) and Use two SVs (μ_1 and μ_2) for US Male Age 65-69



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SVD to Solve $\{a_x\}$, $\{b_x\}$ and $\{k_t\}$,

$$\ln(m_{x,t}) \approx A_x B_t = a_x + b_x \cdot k_t \qquad (6)$$

$$a_x = \frac{1}{T}\sum_t A_x B_t = A_x \left(\frac{\sum_t B_t}{T} \right)$$

$$k_t = \left(\sum_x A_x \right) \cdot \left(B_t - \frac{\sum_t B_t}{T} \right) \qquad (7)$$

$$b_x = \frac{A_x B_t - a_x}{k_t} = \frac{A_x}{\sum_x A_x}$$

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US Results - $\{a_x\}$ and $\{b_x\}$ from equation (7) above, for example,

	a_x		b_x	
Age	Male	Female	Male	Female
5-9	-7.7223	-8.0189	0.0797	0.0750
10-14	-7.6368	-8.1008	0.0787	0.0757
20-24	-6.2366	-7.1491	0.0641	0.0668
30-34	-6.1041	-6.7085	0.0628	0.0627
40-44	-5.4378	-5.9529	0.0560	0.0556
50-54	-4.5814	-5.1329	0.0472	0.0479
60-64	-3.7630	-4.3153	0.0388	0.0403
70-74	-2.9779	-3.4321	0.0307	0.0321
80-84	-2.1463	-2.4754	0.0221	0.0232
90-94	-1.3603	-1.5505	0.0140	0.0145

A second stage of estimation of k_t, whereby the k_t's are recalculated from the equation,

$$D(t) = \sum [N(x,t) \cdot \exp(\hat{a}_x + k_t \cdot \hat{b}_x)] \qquad (8)$$

Taking the estimated $\{a_x\}$ and $\{b_x\}$ as fixed from equation (7).

Note that: there is no closed form solution for equation (8) above. (Newton method is employed)

Take an initial k_{t1} equation along with $\{a_x\}$ and $\{b_x\}$ from equation (7) above, the following vector is employed for the Newton's method to obtained the 2nd stage k

Re-write the equation (8) above,

$$F(T) = D(T) - N(X,T)^{'} \cdot e^{a(X)+k(T)\cdot b(X)} \qquad (9)$$

The Jacobian matrix for equation (9) is,

$$J(T) = -N(X,T) \times b(X) \times e^{a(X)+k(T)\times b(X)} \Delta k(T)$$

The first order Taylor series becomes:

$$D(T_i) - N(X,T_i)^{'} \cdot e^{a(X)+k(T_i)\cdot B(X)} + \left(-N(X,T_i) \times b(X) \times e^{a(X)+k(T_{ii})\times b(X)}\right) \cdot \left(k(T_{i+1}) - k(T_i)\right) \to 0$$

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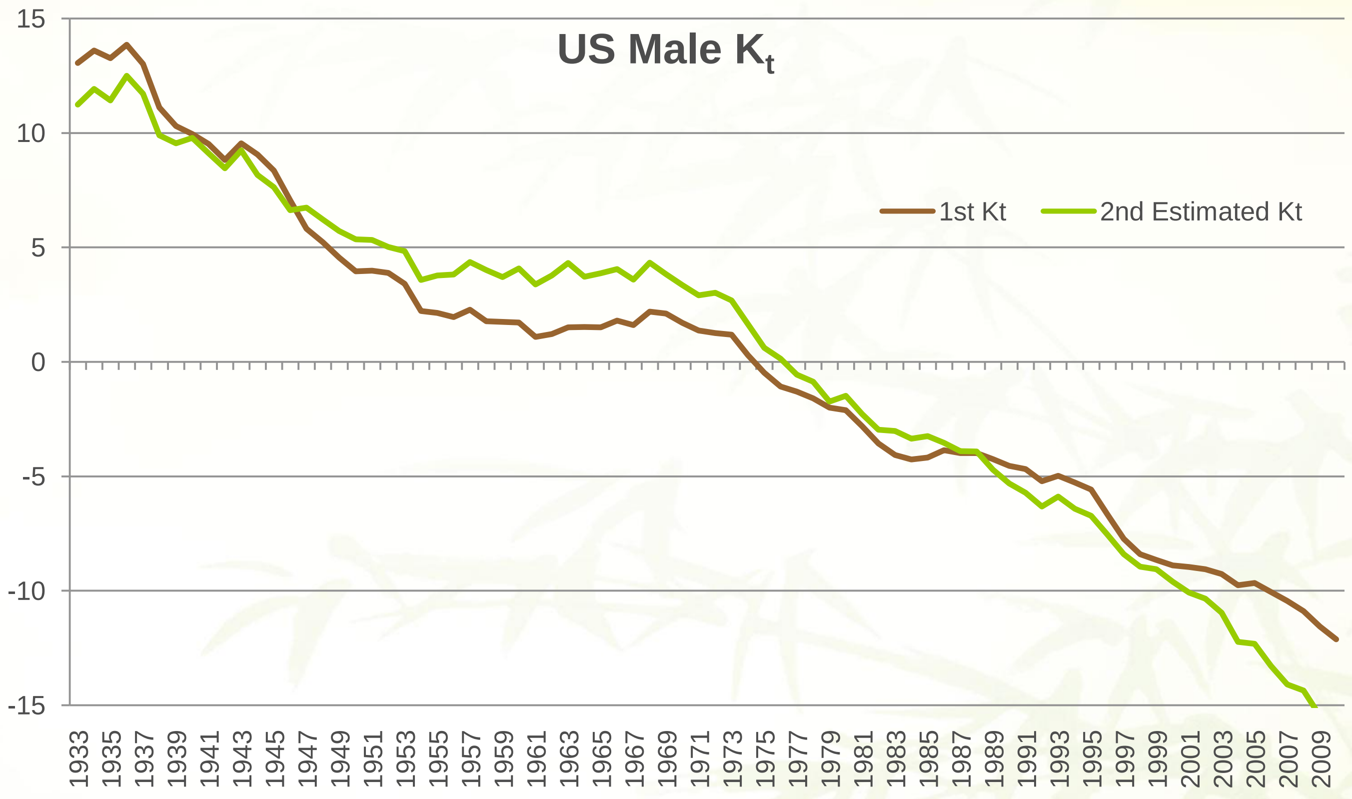
US Male K_{t1} and Re-Estimated K_{t1} based on 1933 -2010



ARIMA(0,1,0) time series model that a random walk with drift is found to be a good fit, for the mortality index k_t, That is,

$$k_t = c + k_{t-1} + u_t \qquad (10)$$

the projection of the k_t into the i years from the current year t,

$$k_{t+i} = k_t + iC + \sum_{j=0}^{i-1} u_{t+j} \qquad (11)$$

Equation (11) implies, the quantity for the error term is,

$$\sqrt{\sum_{j=0}^{i-1} u_{t+j}} = \sqrt{i} \cdot \sigma$$

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From the k_t formula (11),

$$k_t - k_{t-1} = C + u_t$$
$$k_{t-1} - k_{t-2} = C + u_{t-1}$$
$$\vdots$$
$$k_{t-(m-1)} - k_{t-m} = C + u_{t-(m-1)}$$

Summing up the above,

$$C \approx \frac{1}{m}\sum_{j=0}^{m-1}\left(k_{t-j} - k_{t-j-1}\right)$$

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Lee Carter Model Steps:

Initial LC Model

- Model specification

SVD

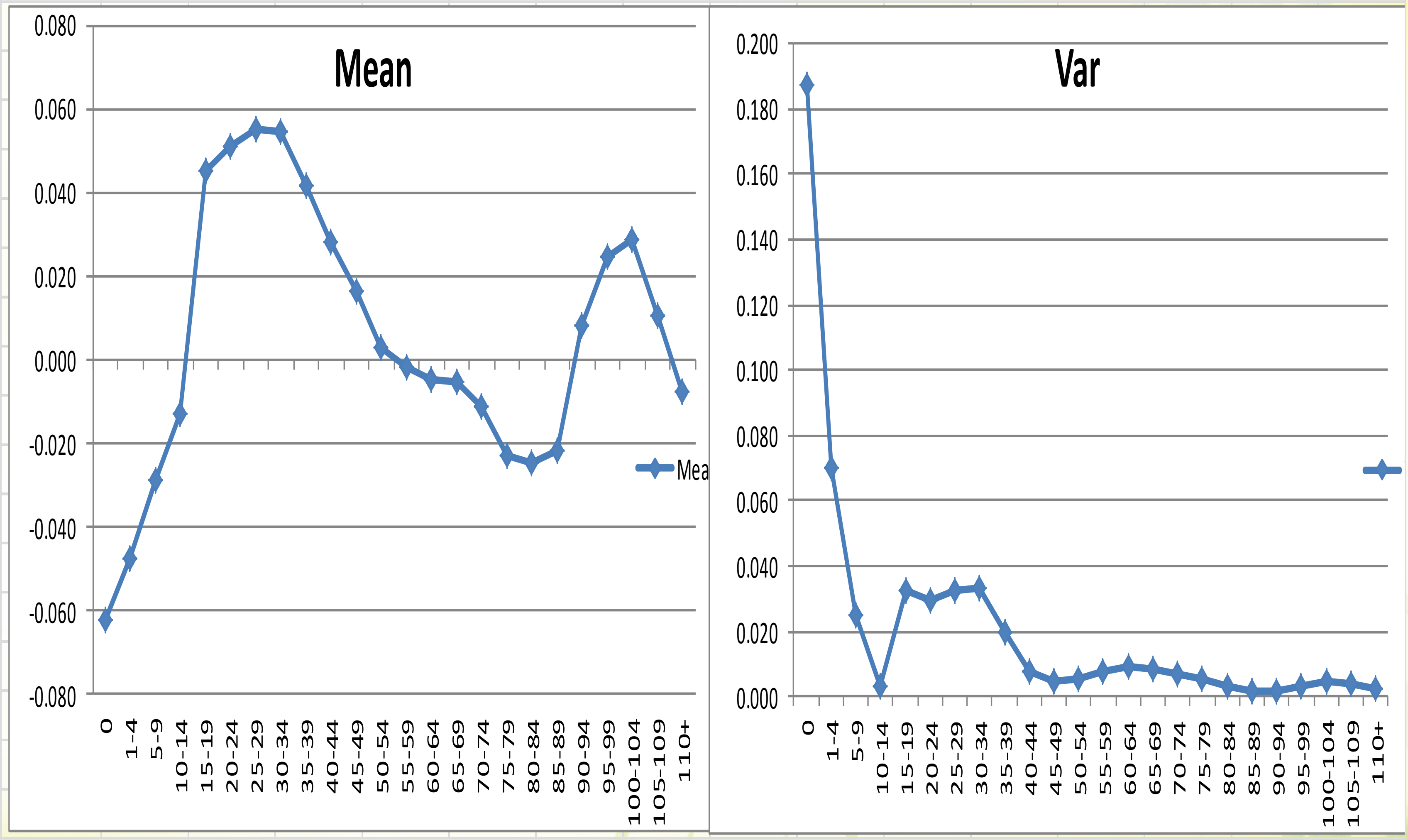
- SVD values
- Preliminary Solution of a_x, b_x, k_t

Model k_t

- Newton's Method Re-estimate k_t
- Random walk with drift

The error Term $\varepsilon_{x,t}$ Analysis

Recall that the error is assumed to follow a normal distribution with mean 0 and standard deviation σ_{ε}. The σ_{ε} should not be too much different across ages.


Mean
Var
0.080
0.060
0.040
0.020
0.000
-0.020
-0.040
-0.060
-0.080
Mea
0.200
0.180
0.160
0.140
0.120
0.100
0.080
0.060
0.040
0.020
0.000
0
1-4
5-9
10-14
15-19
20-24
25-29
30-34
35-39
40-44
45-49
50-54
55-59
60-64
65-69
70-74
75-79
80-84
85-89
90-94
95-99
100-104
105-109
110+

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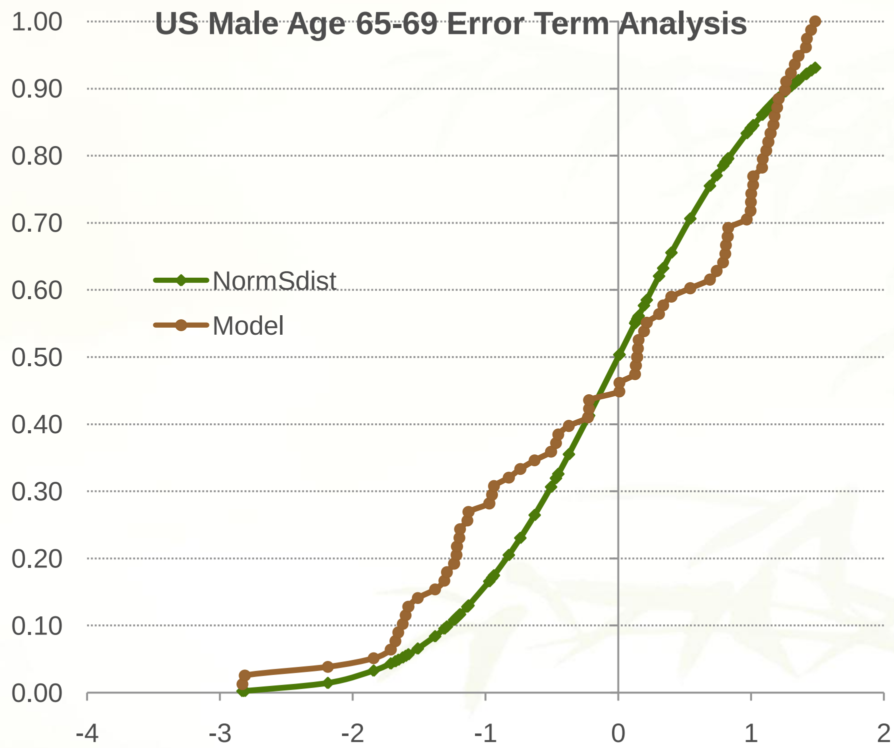

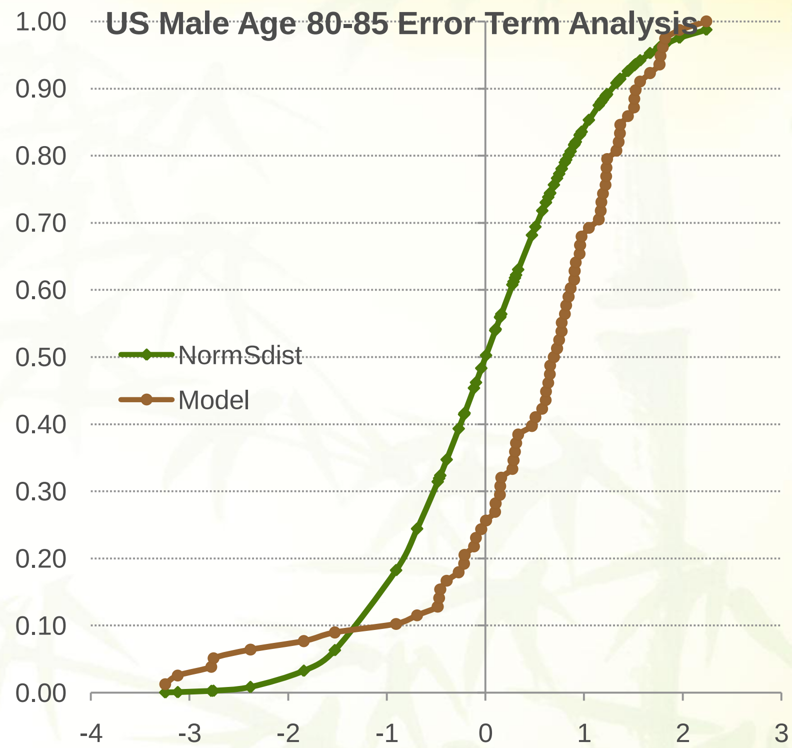

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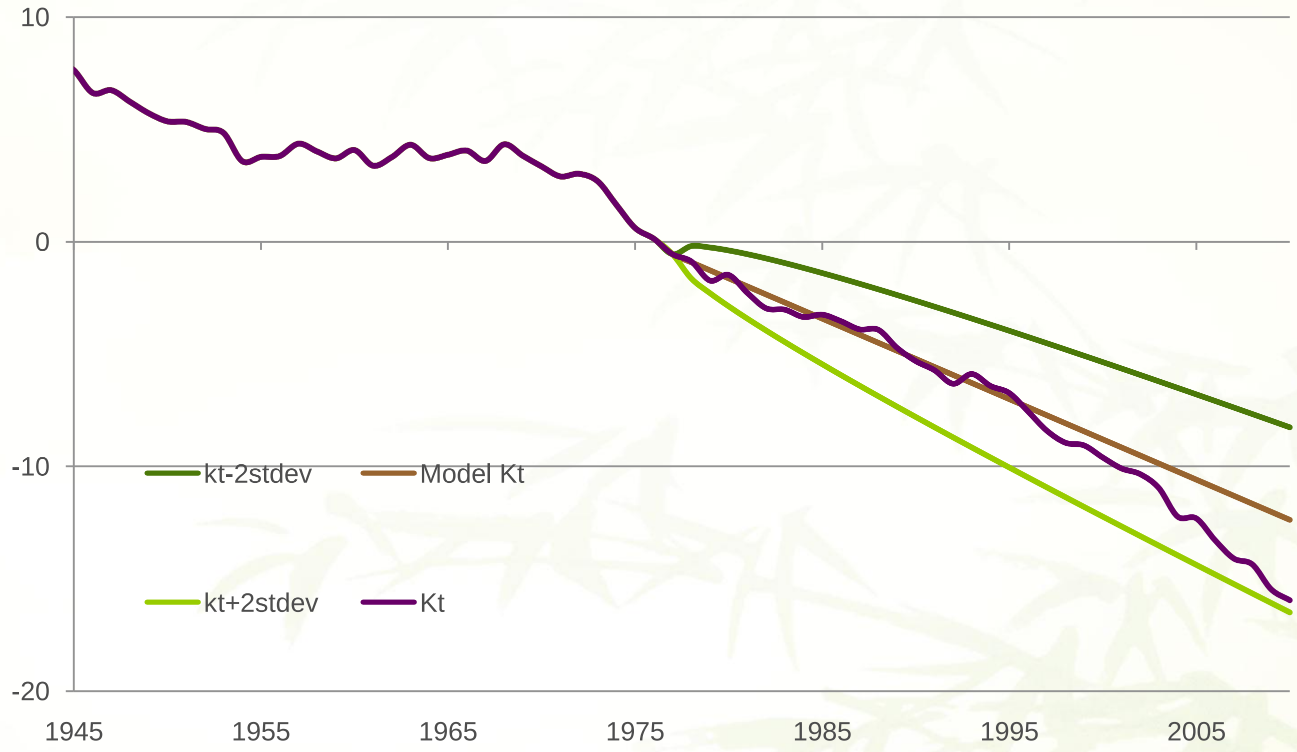
Re-Estimated K_t Projection



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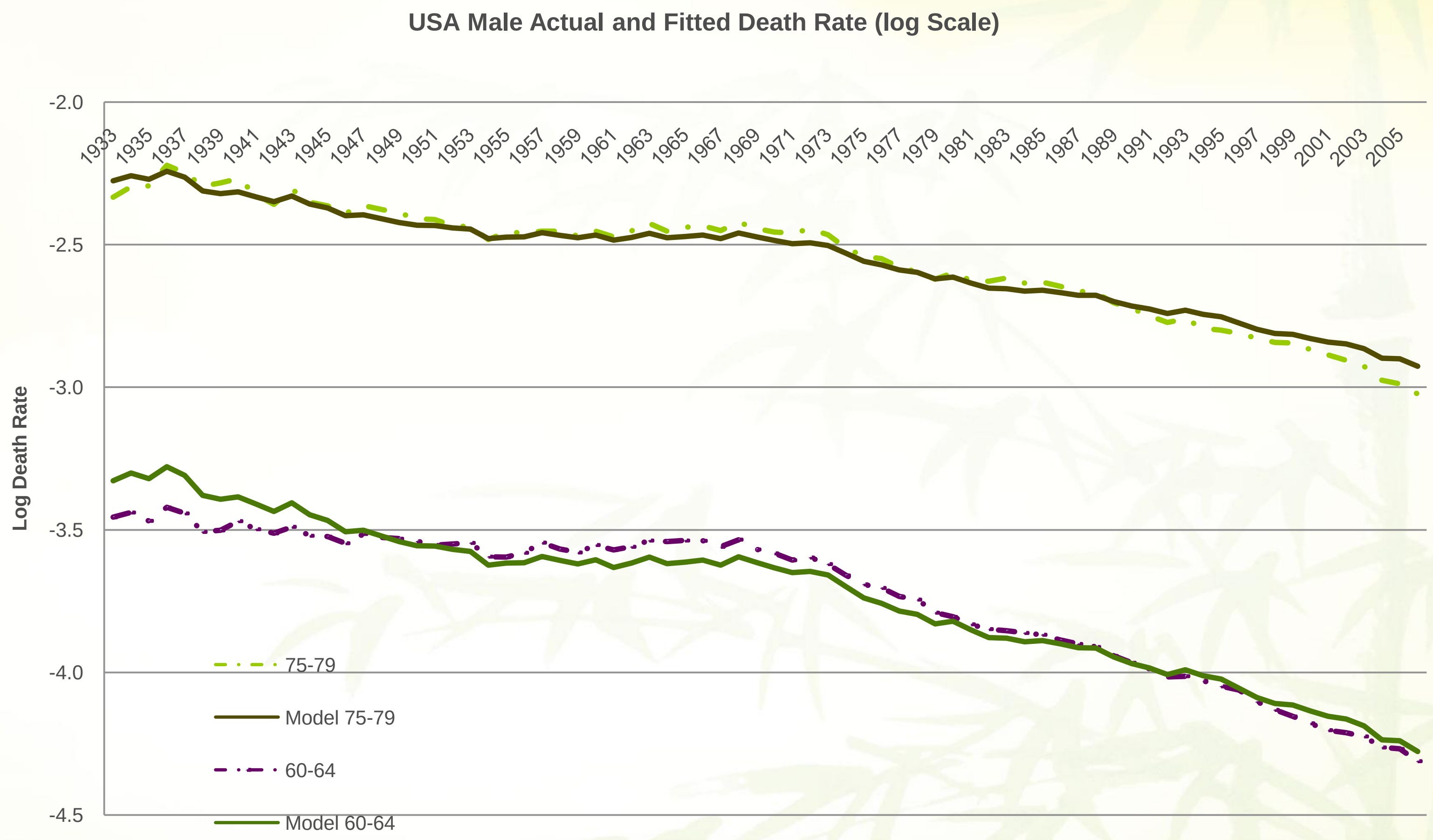

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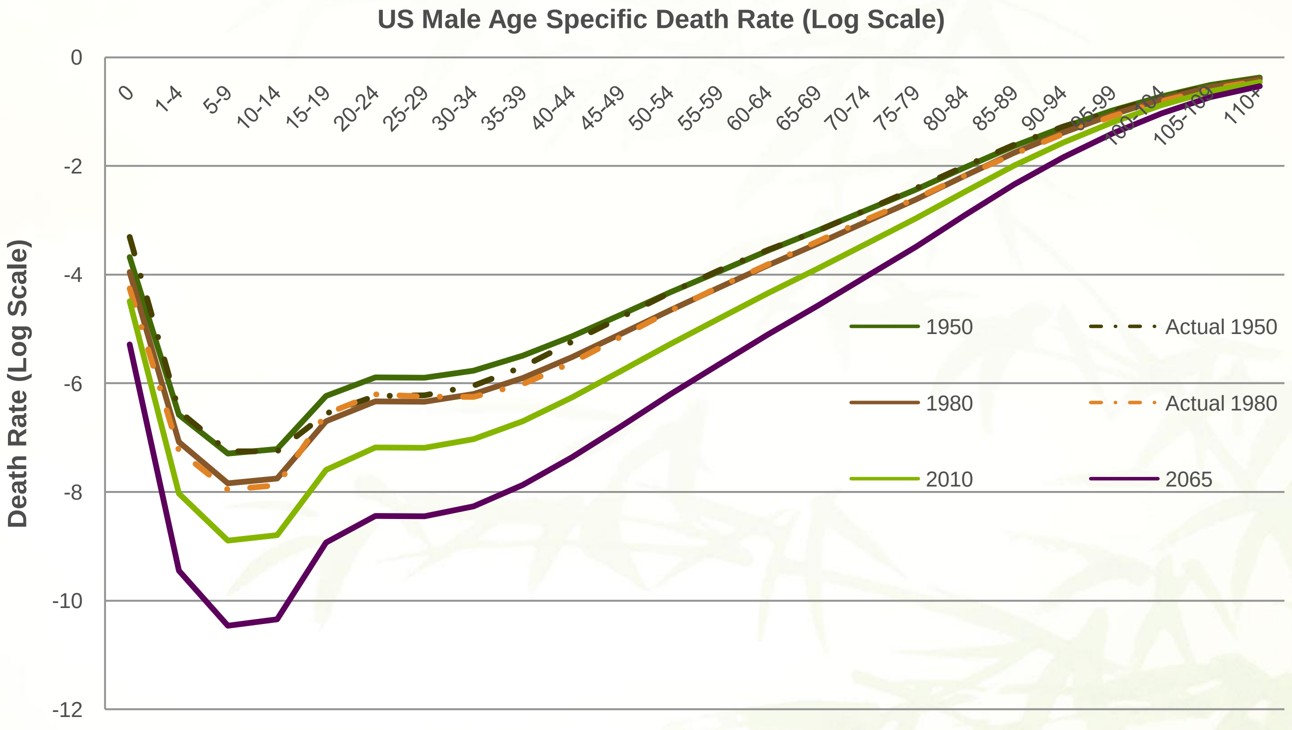

Error Analysis

- Mean and Variance analysis
- Normal distribution approximation

Mort Index k_t

- Reasonable pattern
- with 2 standard deviation

Mortality Rate

- Mortality Rate (log scale) model vs. actual
- Mortality projection Model vs. actual

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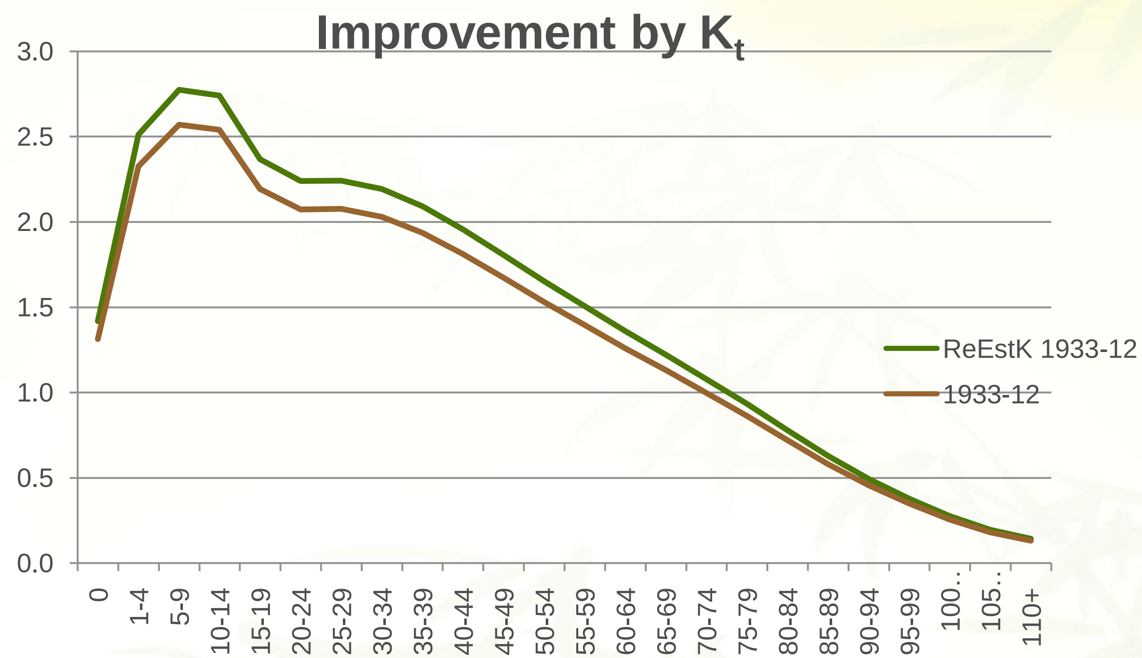

The Re-estimated k_t resulting in higher improvement factor.

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Specification

Mortality Improvement by Age and Region

Comparison

Specification

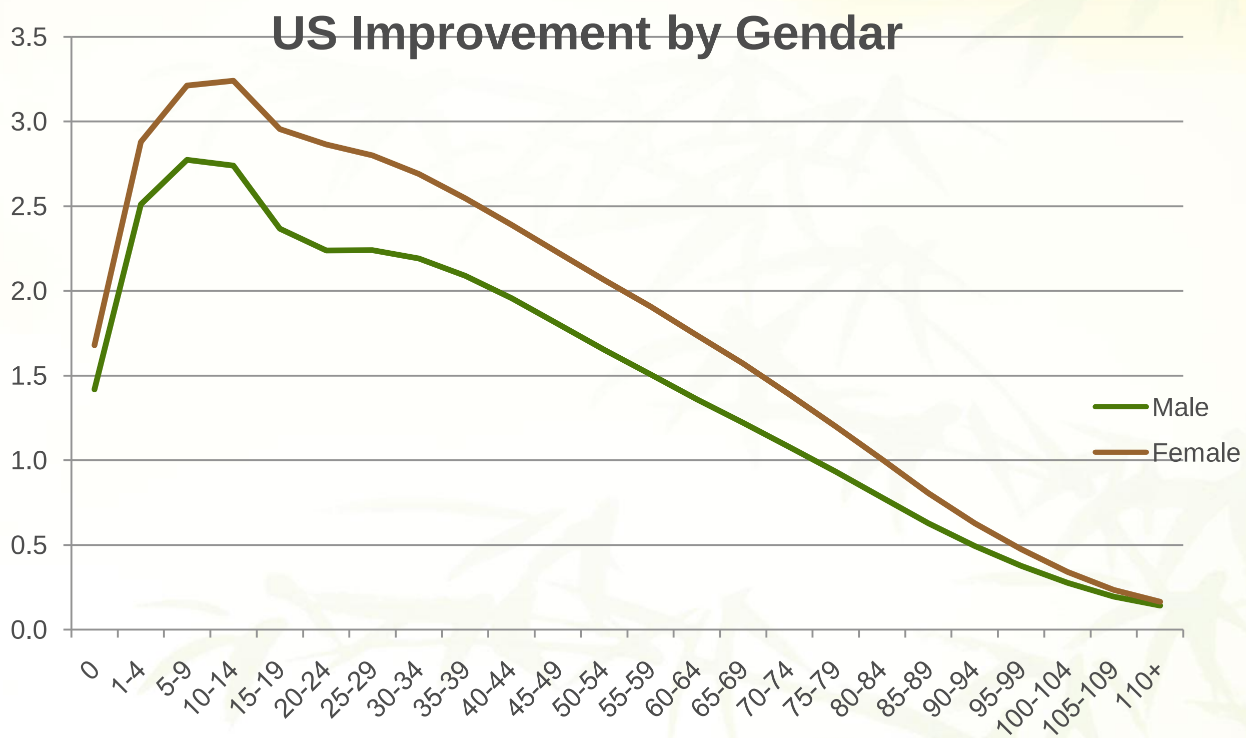
Projection period:

Region	ProjPeriod	# of ProjYr
Japan	1973-2012	40
Taiwan	1970-2010	41
USA	1971-2010	40

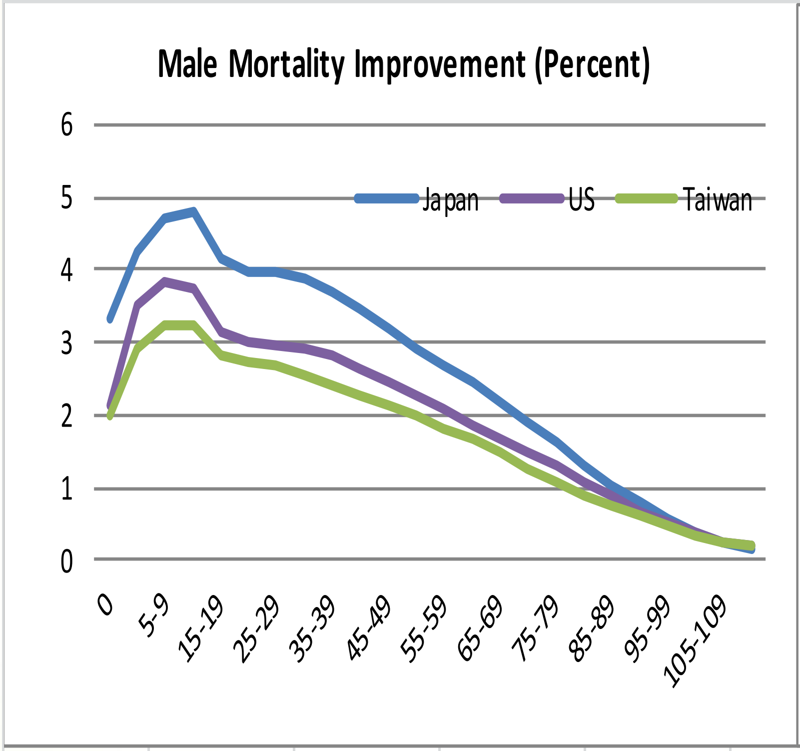
Use the first Singular Value

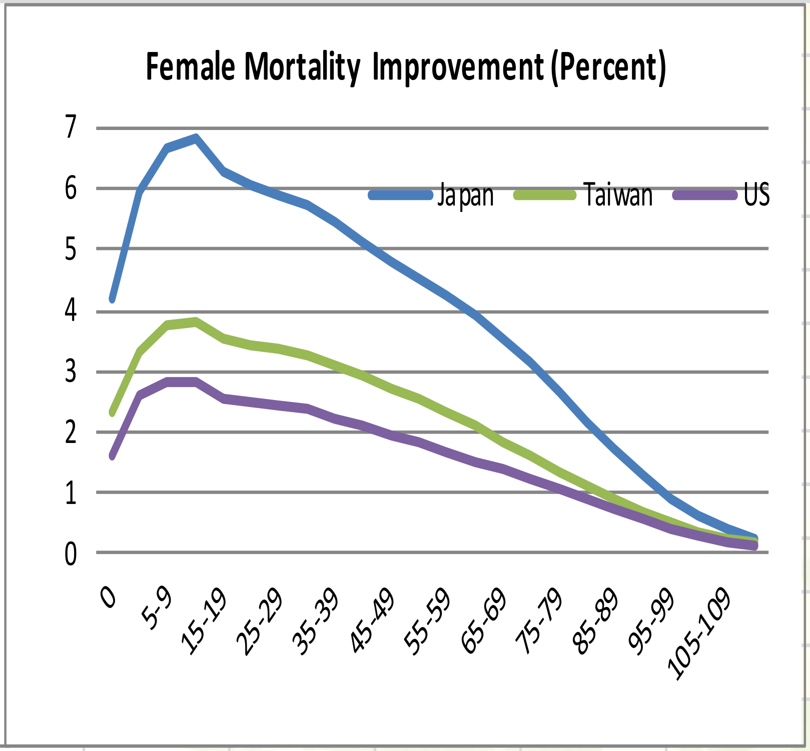
Use Re-estimated k

Age	Male			Female		
	Japan	Taiwan	US	Japan	Taiwan	US
5-9	4.70	3.26	3.85	6.64	3.77	2.85
20-24	4.00	2.74	2.99	6.04	3.42	2.50
30-34	3.89	2.57	2.93	5.72	3.24	2.36
40-44	3.45	2.29	2.65	5.15	2.91	2.09
50-54	2.93	1.99	2.27	4.51	2.52	1.81
70-74	1.91	1.27	1.49	3.14	1.60	1.24
80-84	1.32	0.91	1.08	2.18	1.12	0.91


US Improvement by Gendar
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1-4
5-9
10-14
15-19
20-24
25-29
30-34
35-39
40-44
45-49
50-54
55-59
60-64
65-69
70-74
75-79
80-84
85-89
90-94
95-99
100-104
105-109
110+
Male
Female

Comparison by Region:





Conclusion

1. Lee-Carter Method modeled the uncertainty of the reduction factor while other methods didn't;
2. The reduction factor is vary by age and sex;
3. The reduction factor is also vary by projection year;
4. The reduction factor vary by region;